ITCS 312: Formal Languages and Automata

Final Exam, Second semester 2011/2012, Form: $\boxed{\text{A}}$

Name: ____________________

Student Number: ____________________

Section: ____________________

Section 1. (1 point each)

Mark the following statements with True if they are true and False otherwise.

____ In order for a Turing machine to accept a word w, it must scan all its letters and it must halt in a final state.

____ The language $L = \{a^n b^n a^n b^n : n \geq 1\}$ can be accepted by an NPDA.

____ Let L_1 and L_2 be two regular languages, then $(L_1^* \cap L_2^*)$ is also regular.

____ JFLAP can be used to transform an NFA to a DFA..

____ The set of all languages over an alphabet Σ is defined as 2^{Σ^*}. .

____ Given a set S which is countable and infinite then its power set 2^S is uncountable.

____ Given any nonempty alphabet Σ, there exists languages that are non-recursively enumerable.

____ Every nondeterministic finite automaton M can be converted to a deterministic one M' that accepts the same language.

____ If a language is accepted by a Turing machine then it must be a recursive language.

____ The language $L((aa)^* b^* (aa)^* + bb^* a)$ contains the string $abbaaaaa$.

____ ANTLR can be used to generate lexical analyzers and parser for a given context-free grammar if it does not contain left-recursion.

____ An NPDA can accept any language that is generated by a right-linear grammar.

____ The programming languages Pascal, C++ and Java are all examples of context-free languages.

____ We can show that the language $L = \{w \in \{0,1\}^* : n_0(w) \text{ is even and } n_1(w) \text{ is odd}\}$ is not regular using the pumping lemma.

____ In order to use exhaustive search to find a parsing for a word using a grammar, we only have to eliminate lambda productions from the grammar.

____ By definition, a function f with domain D is Turing-computable if there exists a Turing machine M such that for every $x \in D$, $q_0 x \vdash^* q_f f(x)$, where $q_f \in F$.

____ The function $f(x) = 45x + 17$ with domain being the positive integers is Turing-computable.

____ Turing machines are superior in power to finite state machines and NPDAs and can accept any formal language.

____ There exists a Turing machine which accepts the language $L = \{a^n b^m a^{n+m} : n, m \geq 0\}$.

____ The grammar $S \rightarrow aSb|bSa|SS|A, A \rightarrow aAb|\lambda$ is ambiguous.

Section 2. (5 points each)

1. Classify the following language according to the Chomsky hierarchy. Write the name of the largest class of languages that a language is included in. For example, if the language is regular and context-free, then you should use say it is context-free.

Language	Chomsky Heirarchy Type
$\{a^n b^n c^n : n \geq 0\}$	
$\{a^n : n \geq 10\}$	
$\{w \in \{0, 1\}^* : w$ has an even number of 0's$\}$	
$\{w \in \{a, b\}^* : w$ has more a's than b's $\}$	
Every $w \in \{a, b\}^+$ which do not contain the substring abba	

2. Consider the following finite automaton.

   (a) Convert it to a DFA.

   (b) What is the language accepted by this automaton?

3. Show that the language

$$L = \{w \in \{0,1\}^* : n_1(w) > n_0(w)\}$$

is context free.

4. Consider the following Turing machine: $M = (\{q_0, q_1, q_2\}, \{0,1\}, \Gamma, \delta, q_0, \Box, \{q_5\})$ where the transition function is defined as

$$\begin{array}{rcl}
\delta(q_0, 0) & = & (q_1, x, R) \\
\delta(q_1, 0) & = & (q_1, 0, R) \\
\delta(q_1, y) & = & (q_1, y, R) \\
\delta(q_1, 1) & = & (q_2, y, R) \\
\delta(q_2, 0) & = & (q_3, y, L) \\
\delta(q_3, y) & = & (q_3, y, L) \\
\delta(q_3, 0) & = & (q_3, 0, L) \\
\delta(q_3, x) & = & (q_0, x, R) \\
\delta(q_0, y) & = & (q_4, y, R) \\
\delta(q_4, y) & = & (q_4, y, R) \\
\delta(q_4, \Box) & = & (q_5, \Box, R)
\end{array}$$

(a) What is the language accepted by M?

(b) What are the elements of the set Γ in the Turing-machine?

5. Show that the following language is non-regular using the pumping lemma

$$L = \{a^n b^\ell : n \geq 2\ell\}.$$

6. Construct a Turing machine which will compute the following function $f(m) = 3m$ where m is a positive integer. Note that m is represented on the tape by unary notation.

7. Construct an NPDA for the following language.

$$L = \{w \in \{a,b\}^* : a^{3n}b^n\}$$

8. Find a regular grammar for the following language $L = \{a^n b^m : \ n + m \text{ is even}\}$

9. Find a Turing machine that accepts the language $L = \{a^n b^n c^{2n}\}$

Answer Key for Exam $\boxed{\mathsf{A}}$

Section 1. (1 point each)

Mark the following statements with True if they are true and False otherwise.

False In order for a Turing machine to accept a word w, it must scan all its letters and it must halt in a final state.

False The language $L = \{a^n b^n a^n b^n : n \geq 1\}$ can be accepted by an NPDA.

True Let L_1 and L_2 be two regular languages, then $(L_1^* \cap L_2^*)$ is also regular.

True JFLAP can be used to transform an NFA to a DFA..

True The set of all languages over an alphabet Σ is defined as 2^{Σ^*}. .

True Given a set S which is countable and infinite then its power set 2^S is uncountable.

True Given any nonempty alphabet Σ, there exists languages that are non-recursively enumerable.

True Every nondeterministic finite automaton M can be converted to a deterministic one M' that accepts the same language.

False If a language is accepted by a Turing machine then it must be a recursive language.

False The language $L((aa)^*b^*(aa)^* + bb^*a)$ contains the string $abbaaaaa$.

True ANTLR can be used to generate lexical analyzers and parser for a given context-free grammar if it does not contain left-recursion.

True An NPDA can accept any language that is generated by a right-linear grammar.

True The programming languages Pascal, C++ and Java are all examples of context-free languages.

False We can show that the language $L = \{w \in \{0,1\}^* : n_0(w) \text{ is even and } n_1(w) \text{ is odd}\}$ is not regular using the pumping lemma.

False In order to use exhaustive search to find a parsing for a word using a grammar, we only have to eliminate lambda productions from the grammar.

True By definition, a function f with domain D is Turing-computable if there exists a Turing machine M such that for every $x \in D$, $q_0 x \vdash^* q_f f(x)$, where $q_f \in F$.

True The function $f(x) = 45x + 17$ with domain being the positive integers is Turing-computable.

False Turing machines are superior in power to finite state machines and NPDAs and can accept any formal language.

True There exists a Turing machine which accepts the language $L = \{a^n b^m a^{n+m} : n, m \geq 0\}$.

True The grammar $S \to aSb|bSa|SS|A, A \to aAb|\lambda$ is ambiguous.

Section 2. (5 points each)

1. Classify the following language according to the Chomsky hierarchy. Write the name of the largest class of languages that a language is included in. For example, if the language is regular and context-free, then you should use say it is context-free.

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\end{aligned}$$

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